Fluidized-bed Heat Transfer Correlation

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In the March, 1958, issue of the *A. I. Ch. E. Journal* Wender and Cooper published a fluidized-bed heat transfer correlation which is open to question on a number of counts. It is the purpose of this discussion to elaborate on a few of these aspects.

It must be regretted that the authors did not include the data of van Heerden and coworkers in their analysis. Some of the exterior-wall–fluidized-bed heat transfer studies more frequently referred to are indicated in the accompanying tabulation, and it is readily seen that the van Heerden study is the most comprehensive. The experiments were carefully designed to disclose the complicated effects of the system variables, and the quality of the work and its accuracy are high. Hence much would have been gained had the authors chosen to use these data.

As the reason for omitting the van Heerden data from their analysis the authors cited the fact that van Heerden and co-workers did not accompany their data with bed-voidage values. This reason is however difficult to accept, especially as the final correlation of the authors contains a bed-voidage term, which must somehow be predictable in advance if their correlation is to have any application at all. There are several methods which permit estimation of the bed voidage with quite good results, and it is hard to conceive that the authors were not aware of some of these. One of the methods was published as early as 1951 (1). Over the years it has been thoroughly tested and found to apply to many sets of data quite satisfactorily. It has therefore been applied to the van Heerden data in order to examine what agreement there is between these and the exterior-wall–bed correlation of the authors. The comparison is given in the accompanying graph. Merely three sets of data have been selected for this demonstration. The solids chosen are all nonvesicular; hence there is little likelihood that the calculated voidages are in substantial error. But even if these voidages were in error by, say, 1 to 5 percentage points, as they conceivably could in the limit be, the magnitude of the calculated heat transfer coefficients and their path would not be greatly different. In addition to the data shown in Figure 1 other runs pertaining to a number of other solids and gases were examined similarly, and the comparisons with the proposed correlation were of the same order as is shown by these selected data.

Considering first comparison 1, one sees a fine agreement at $Re = 0.1$; however at $Re = 2.0$ van Heerden reported an experimental coefficient of about 75 B.t.u./(hr.)(sq. ft.)(°F.), whereas the correlation predicts a value of nearly 200 B.t.u./(hr.)(sq. ft.)(°F.) This is a very severe deviation in a wrong and unsafe direction, as far as application to design is concerned. The reason for the deviation is of course that the correlation does not incorporate properly the functional relationship between fluid mass velocity and bed expansion.

In comparison 2 a solid of essentially similar physical properties and virtually the same particle diameter has been fluidized, but with a gas of much greater thermal conductivity and lesser density. First one observes that the course of the calculated data is still the same as in comparison 1, and hence at variance with the experimental values. It appears in addition that now the calculated data are for the greater part much lower than the experimental data. This severe deviation leads to the conclusion that the correlation apparently does not properly account for the thermal and any other physical properties of the fluidizing fluid.

A third comparison is shown which pertains to a larger solid, fluidized at a higher range of Reynolds numbers. Here again the calculated data are unduly high, though the course of the calculated data is now in agreement with the path of the experimental values. As a matter of general interest the data predicted by our generalized correlation are shown (*2*).

In closing this discussion, we appreciate the immense amount of tedious calculation procedure, plotting, and cross plotting which is associated with this working method. However, though the numbers of runs covered and cited in support of a correlation may be quite impressive, it is doubtless of more value to point out the reasons why some particular data points may fail to agree, simply because such lack of agreement may hold the key to a basic understanding of the phenomenon. Unfortunately this has not been achieved in this work to the extent which would have been desirable and constructive. It must be borne in mind that apparatus details, such as gas-inlet devices, hence entrance effects, exposed bed sections, temperature-measuring devices, and other internals in the bed must affect fluidization performance and therefore the heat transfer coefficients.

Little if anything at all will be said about the internal surface-bed correlation. It is of an entirely different form and type from the exterior-wall–bed correlation. Nevertheless both expressions refer to one and the same phenomenon, namely fluidization, occuring though under somewhat different conditions. The question may rightly be asked whether in the light of two such different formulations as have been proposed, a unified correlation for both external as well as internal surface data is possible.

LITERATURE CITED

1. Leva, Max, *et al.*, *U. S. Bur. Mines, Bull. No. 504* (1951).
2. Wen, C. Y., and Max Leva, *A. I. Ch. E. Journal*, 2, 482 (1956).

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EXPERIMENTAL VARIABLES, DENSE PHASE DATA

Investigator	Materials	D_p - feet	Solids Density lbs/ft^3	Fluids
van Heerden et al	Carborundum, Iron Oxide, Coke, Lead, Fly Ash, Dev Alloy	0.000262 to 0.00213	112 — 694	Air, Argon, CH_4 CO_2, Town Gas H_2+N_2 mixtures
Dow-Jakob	Aerocat, Coke, Iron Powder	0.000252- 0.000560	121 — 466	Air
Leva et al	Sand, Fe-Catalyst Silica Gel	0.000129- 0.00149	80 — 500	Air, CO_2, Helium
Toomey — Johnstone	Glass Beads	0.000179 to 0.00278	167 — 179	Air
Bartholomew-Katz	Sand, Aluminum Powder, $CaCO_3$	0.000277- 0.000822	160 — 167	Air

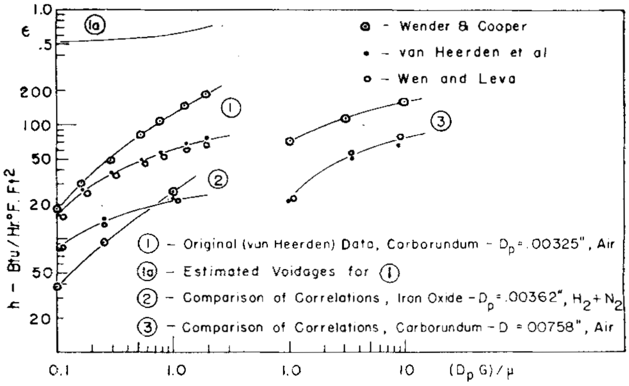

Fig. 1. Some typical data of van Heerden, and how they are predicted by the correlation of Wender and Cooper.

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This means that the approximate solution can be used in a wider range. When the Graetz number becomes smaller, the average Nusselt number becomes

$$\left(\frac{h_M D}{\lambda}\right)_{(WC_p/\lambda l)\to 0} = \frac{2a\alpha}{(1-a)^2}\beta_0^{\,2} = 4.94 \quad (4.11)$$

When $a = 0.5$, the average Nusselt number is not theoretically existent below 4.94.

Example

Average bulk temperature and average temperature at the exit of the pipe, when synthetic resin powder at 20°C. is extruded by the piston at the rates of 2.0 and 0.2 cm./sec., are computed. The resin has the following characteristics: heat conductivity = 0.14 kcal./meter-hr. (°C.), specific heat = 0.4 kcal./(kg.)(°C.) and density = 1,400 kg./cu. meter. The dimensions of the pipe are 1 meter in length and 3 cm. in diam. The pipe is heated externally and the wall temperature changed linearly with the axis, if it is assumed that 150°C. at the inlet and 200°C. at the exit are maintained.

Solution

This example is a practical case when synthetic resin is molded. In this case the information in the section on theoretical solution when flow is rodlike and wall temperature is not constant can be applied; thus $t_1 = 20$°C., $t_w = 150$°C., and $t_{wl} = 200$°C. From Equation (3.3) $C = 1/3$. Graetz numbers are

$$WC_p/\lambda l = 203.55 \quad \text{at} \quad \bar{u} = 2.0 \text{ cm./sec.}$$

$$WC_p/\lambda l = 20.35 \quad \text{at} \quad \bar{u} = 0.2 \text{ cm./sec.}$$

The bulk temperature at the exit is obtained from Equation (3.5)

$$\theta_{pl0} = 2\sum_{s=1}^{\infty} e^{-\pi\psi_s^2(\lambda l/WC_p)}/\psi_s J_1(\psi_s) + \frac{ct_w}{t_1 - t_w}\frac{l}{\pi(\lambda l/WC_p)}\left[\pi\left(\frac{\lambda l}{WC_p}\right) - \frac{1}{4} + 2\sum_{s=1}^{\infty} e^{-\pi\psi_s^2(\lambda l/WC_p)}/\psi_s^3 J_1(\psi_s)\right]$$

In the case of $\bar{u} = 0.2$ cm./sec. from the table of Bessel function

$$\begin{aligned}
\psi_1 &= 2.40483 & J_1(\psi_1) &= 0.51915\\
\psi_2 &= 5.52008 & J_1(\psi_2) &= -0.34026\\
\psi_3 &= 8.65374 & J_1(\psi_3) &= 0.27145\\
\psi_4 &= 1.17915 & J_1(\psi_4) &= -0.23246\\
&\vdots & &\vdots
\end{aligned}$$

By substituting these values in this equation

$$\begin{aligned}
\theta_{pl0} &= (t_{pl0} - t_w)/(t_1 - t_w)\\
&= 2\{3.2825 \times 10^{-1} - 4.832 \times 10^{-3} + 7.109 \times 10^{-6} - \cdots\}\\
&\quad - 2.4924[-9.569 \times 10^{-2} + 2\{5.676 \times 10^{-2} - 1.588 \times 10^{-4} + 9.49 \times 10^{-8} - \cdots\}]\\
&= 0.6318
\end{aligned}$$

Therefore

$$t_{pl0} = 71.59°\text{C.}$$

The average temperature at the exit can be calculated from Equation (3.8), and the result is shown in Table 4. The bulk temperature is not different from the inlet temperature, even at Graetz number = 203, as shown in Table 4.

When the wall temperature is constant, θ_{Ml} can be obtained either from Figures 4 and 5 or from the equations in the section on theoretical solution when flow is rodlike and wall temperature is not constant by substituting $C = 0$. The calculated result is also given in Table 4. This shows that the average Nusselt number and the average coefficient of heat transfer at the constant wall temperature are considerably smaller than that in the case of increasing wall temperature along the y axis.

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CONCLUSION

The results of this study show that the logarithmic mean of the difference between average bulk temperature and wall temperature can be used in the calculations of average Nusselt number or average heat transfer coefficient. The average bulk temperature at the exit is given in Equation (1.44). When $a = 0.5$, the average bulk temperature can be calculated by Equation (4.5).

Figure 5 shows the relationship between the Nusselt number and Graetz number. From both theoretical and approximate solutions the following equation was derived:

$$\frac{h_M D}{\lambda} = 1.75\left\{\left(\frac{WC_p}{\lambda l}\right)\left(\frac{1-a}{4a\alpha}\right)\right\}$$

when

$$\{(WC_p/\lambda l)\cdot(1 - a)/(4a\alpha)\} > 100$$

When wall temperature changes linearly along the y axis and velocity distribution is rodlike flow, the logarithmic mean can be used as the average-temperature difference. It is assumed that the logarithmic mean can be used even in the case of laminar flow, where the velocity distribution is not the same as it is in rodlike flow.

BOOKS

Viscous Flow Theory I, Laminar Flow, Shih-I Pai, D. Van Nostrand Company, Inc. 384 pages.

The development of aircraft and missiles which travel at velocities in excess of the velocity of sound has necessitated much new research in the hydrodynamics of compressible fluids. For though it is possible to neglect the compressibility of air at low speeds (circa 200 miles per hour), this is not possible at higher speeds. The book under review is concerned with the laminar flow of viscous, compressible fluids with special attention to aerodynamics. Three major topics are discussed: (a) the classical hydrodynamic theory of fluids, including some elementary kinetic theory of gases; (b) generalizations derivable from the theory without explicit solution of the differential equations, such as similarity and dimensional analysis and general properties of the Navier-Stokes equation; and (c) boundary layer theory. The latter is by far the largest section, occupying some 216 pages. Considerable detail is given, and numerous tables of useful data are included in the text. Turbulent flow is treated in part II of this work.

This reviewer feels that the major omission from the text is a discussion of the properties of gases at extremely low pressures. Under circumstances prevailing in the upper atmosphere the mean free path of a molecule may easily be of the order of magnitude of the dimensions of the flying object. Under these conditions the relative

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